Astrophysical Sources of Stochastic Gravitational Radiation in the Universe

K.A.Postnov

Sternberg Astronomical Institute, Moscow State University, Moscow 119899, Russia

Abstract

Stochastic gravitational waves (GW) associated with unresolved astrophysical sources at frequency bands of the ongoing GW interferometers LIGO/VIRGO and LISA are studied. We show that GW noise from rotating galactic neutron stars with low magnetic fields may reach the advanced LIGO sensitivity level at frequency $f \sim 100$ Hz. Within LISA frequency band ($10^{-4} - 10^{-1}$ Hz), the GW background from galactic binary stars is shown to mainly contribute up to a frequency of 3×10^{-2} Hz, depending on the galactic rate of binary white dwarf mergers. To be detectable by LISA, relic GW backgrounds should be as high as $\Omega_{GW} h_{100}^2 > 10^{-8}$ at 10^{-2} Hz.

1 Introduction

In a few years, with the completion of construction of gravitational wave detectors of high sensitivity, a new window into the Universe will be open (see Thorne 1995, Schutz 1996 for a recent review of gravitaitonal wave astronomy). In this connection, extensive studies of possible sources of gravitationa radiation are now being conducted. The most promising targets for the initial laser interferometers with the rms sensitivity level $h_{rms} \approx 10^{-21}$ at $f = 100$ Hz are coalescing binary neutron stars and/or black holes, which can be observable from distances up to 100 Mpc (Lipunov, Postnov & Prokhorov 1997).

Stars are most numerous baryonic objects in the Universe ($\sim 10^{11}$ within the Galaxy, $\sim 10^{21}$ within the Hubble radius $R_H \sim 3000 h_{100}^{-1}$ Mpc, where $h_{100} = H_0/100$ km/s/Mpc is the present value of the Hubble constant). Among them the most significant sources of GW are rotating triaxial neutron stars (NS) and binary stars. Only a small fraction of galactic NS (about 700) is observed as radiopulsars, so when trying to search for GW from them we have an advantage of knowing the precise spin period and the position on the sky. The same argument relates to the binary stars with known orbital periods. But the vast majority of them are unresolved sources and will form a stochastic background. A stochastic GW background is commonly measured in terms of the energy density per logarithmic frequency interval related to the critical energy density to close the Universe, $\Omega_{GW} = dE_{GW}/d\ln f/\rho_{cr}c^2$ ($\rho_{cr} = 3H_0^2/8\pi G \approx 1.9\times10^{-29} h_{100}^2$ g cm^{-3} where c is the speed of light). For comparison with

dimensionless detector's sensitivity h, one commonly uses the equivalent characteristic strain $h_c(f) = (1/2\pi)(H_0/f)\Omega_{GW}^{1/2}$.

Being interesting by themselves, astrophysical backgrounds, however, are viewed as a noise burying a possible cosmological gravitational wave background (CGWB), which bears the unique imprint of physical processes occurring at the very early (near-Plankian) age of the Universe (see e.g. Grishchuk 1988 for a review). Exact value of CGWB is still very controversial (see Grishchuk 1996 for fresh estimates). What is more reliable (however, not completely parameter-free) are GWBs formed by known astrophysical sources (old neutron stars, binary stars), and here we address the question how much astrophysical GW noises contribute at LIGO/VIRGO (10-1000 Hz) and LISA ($10^{-4} - 10^{-1}$ Hz) frequencies.

2 GW noise from sources with changing frequency

To calculate GW noise produced by some unresolved sources we need to know the number of sources per logarithmic frequency interval. At the first glance, this would require knowing precise formation and evolution of sources. However, when only GW carries away angular momentum from the emitting source, the problem becomes very simple and physically clear.

GW energy loss leads to changing the frequency $\omega = 2\pi f$ of emitting objects. In the case of a rotating triaxial body, the positive rotational energy $E_{rot} = I\omega_{rot}^2/2$ (here I is the moment of inertia) is being lost and the spin frequency (hence, GW freqency) decreases. In contrast, in the case of a binary star with masses of the components M_1 AND m_2 in a circular orbit ofr adius a the negative orbital energy, $E_{orb} = -M_1M_2/2a \sim \mathcal{M}c^2 f_{orb}^{2/3}$ (here $\mathcal{M} = (M_1+M_2)^{2/5}(M_1M_2)^{3/5}$ is the so-called "chirp mass" of the system) is being lost and the orbital frequency increases.

To a very good approximation, the conditions of star formation and evolution of astrophysical objects in our Galaxy may be viewed as stationary. This is true at least for last 5 billion years. Let the formation rate of GW sources be $\mathcal{R}$. For example, the mean formation rate of massive stars (>10 $M_\odot$ to produce NS) is about 1 per 30 years.

The stationarity implies that the number of sources per unit logarithmic frequency interval is

$$dN/d\ln f \equiv N(f) = \mathcal{R} \times (f/\dot{f})\,. \tag{1}$$

The total energy emitted in GW per second per unit logarithmic frequency interval at f by all such sources in the galaxy is

$$dE_{GW}/(dt\,d\ln f) \equiv L(f)_{GW} = \widetilde{L}(f)_{GW}N(f) = \widetilde{L}(f)_{GW}\mathcal{R} \times (f/\dot{f})\,, \tag{2}$$

where $\widetilde{L}(f)_{GW}$ is the GW luminosity of the typical source at frequency f ($\widetilde{L}(f)_{GW} \propto f^6$ for non-axisymmetric neutron stars, $\widetilde{L}(f)_{GW} \propto f^{10/3}$ for binary systems).

Finally, for an isotropic background we have

$$\Omega_{GW}(f)\rho_{cr}c^2 = L(f)_{GW}/(4\pi c\langle r\rangle^2) \tag{3}$$

where $\langle r\rangle$ is the inverse-square average distance to the typical source. Strictly speaking, this distance (as well as the binary chirp mass $\mathcal{M}$ and moment of inertia I of NS) may be a function of frequency since the binaries characterized by different $\mathcal{M}$ may be differently distributed in the galaxy. We are highly ignorant about the real distribution of old NS and binaries in the galaxy, but taking the mean photometric distance for a spheroidal distribution in the form $dN \propto \exp[-r/r_0]\exp[-(z/z_0)^2]$ (r is the radial distance to the galactic center and z the hight above the galactic plane) with $r_0 = 5$ kpc and $z_0 = 4.2$ kpc with $\langle r\rangle \approx 7.89$ kpc is sufficient for our purposes.

For the cases considered the energy reservoir radiated in GW is either rotational energy (neutron stars) or orbital energy (binary systems), both depending as some power of the corresponding frequency: $E \propto f^\alpha$, $\alpha_{NS} = 2$, $\alpha_{bs} = 2/3$. Hence the frequecy change $(f/\dot{f})$ may be found from the equation $dE/dt = \alpha E(\dot{f}/f)$ By energy conservation law $dE/dt = (dE/dt)_{GW} + (dE/dt)_{EM} + (dE/dt)_{\ldots}$ where index EM stands for elecromagnetic losses and ... means other possible losses of energy. Finally, we obtain

$$(f/\dot{f}) = \alpha E/((dE/dt)_{GW} + (dE/dt)_{EM} + (dE/dt)_{\ldots}) \tag{4}$$

and

$$L(f)_{GW} = \mathcal{R}\alpha\widetilde{E}\frac{1}{1+\frac{(dE/dt)_{EM}}{(dE/dt)_{GW}}+\frac{(dE/dt)_{\ldots}}{(dE/dt)_{GW}}} \tag{5}$$

The remarkable result is that if GW is the dominant source of energy removal, the resulting GW stochastic background depends only on the source formation rate:

$$\Omega_{GW}(f)\rho_{cr}c^2 = \mathcal{R}\alpha\widetilde{E}/(4\pi c\langle r\rangle^2) \tag{6}$$

3 GWB from old neutron stars at LIGO/VIRGO frequencies

Spin evolution of rotating non-axisymmetric NS with ellipticity ϵ and magnetic moment μ may be driven by GW or elecgtromagnetic losses. The condition that a stochastic signal appears within the detector band deoends on the rate of the frequency change. The upper frequency of the stochastic background for pure electromagnetic energy losses is $f_0^{EM} \approx 10^3(\mathrm{Hz})\mathcal{R}_{30}^{1/2}I_{45}^{1/2}\mu_{30}^{-1}$ where $\mu_{30} = \mu/(10^{30}\mathrm{G\ cm}^3)$ is NS magnetic moment. For pure GW losses this upper frequency is $f_0^{GW} \approx 1.4 \times 10^4(\mathrm{Hz})\mathcal{R}_{30}^{1/4}I_{45}^{-1/4}\epsilon_{-6}^{-1/2}$ where $\epsilon_{-6} = \epsilon/10^6$ is the NS ellipticity.

For plausible values of the NS magnetic fields ($\mu_{30} = 10^{-4}$–10^2) and ellipticities ($\epsilon_{-6} = 10^{-3}$–10^2), at any frequency $< 10^3$ Hz we deal with stochastic backgrounds

from galactic NS. Physically, this is due to the inability of old NS to leave frequency interval $\Delta\omega \sim \omega$ during the typical time between consecutive supernova explosions.

For purely GR-driven NS spin-down the resulting spectrum is independent of the unknown value of ϵ in the NS population. Any additional braking mechanism always lowers the resulting signal. Taking typical values $I = 10^{45}$ g cm^2, $\mathcal{R} = 1/30$ yr^{-1} we obtain from Eq. (6)

$$\Omega_{NS} \approx 10^{-7}\mathcal{R}_{30}^{1/2} I_{45}(f/100Hz)^2 h_{100}^{-2}(r/10kpc)^{-2} \tag{7}$$

or in terms of h_c

$$h_c \approx \frac{1}{\tilde{r}}\sqrt{GI\mathcal{R}/c^3} \approx 10^{-24}\left(\frac{10\text{kpc}}{\tilde{r}}\right)\mathcal{R}_{30}^{1/2} I_{45}^{1/2} \tag{8}$$

(here we assumed the characteristic distance to NS population of order 10 kpc). Remarkably, this limit does not depend on frequency. The GR background of such strength could be detected by the advanced LIGO/VIRGO interferometers in one year integration (Thorne 1987; Giazotto 1997).

For realistic NS parameters the ratio of electromagnetic to GW losses $x = \dot{E}_{EM}/\dot{E}_{GW}$ is

$$x \approx 4000\mu_{30}^2\epsilon_{-6}^{-2}\left(\frac{100\text{Hz}}{f}\right)^2 \tag{9}$$

and electormagnetic losses becomes insignificant ($x \ll 1$) only at high frequencies $f > f_{cr} \approx 6.3(\text{kHz})\,\frac{\mu_{30}}{\epsilon_{-6}}$ If we would take $\epsilon_{-6} = 10^{-3}$ and $\mu_{30} = 10^{-4}$ as in millisecond pulsars, we would obtain $f_{cr} \approx 630$ Hz, however millisecond pulsars are spun up by accretion in binary systems and are not considered here.

Therefore, for realistic NS we must consider the case $x \gg 1$. Then the stochastic background from old NS becomes

$$h_c(f) \approx 5\times 10^{-28}\left(\frac{10\text{kpc}}{\tilde{r}}\right)\mathcal{R}_{30}^{1/2} I_{45}^{1/2}\epsilon_{-6}\mu_{30}^{-1} f \tag{10}$$

and is lis below even advanced LIGO sensitivity at $f \sim 100$ Hz.

We have shown that if the NS form ellipticity is present, the stochastic GR background produced by old NS population is naturally formed due to NS rotation braking. In the limiting case when only GR angular momentum loss causes NS spin-down, this background is *independent* on both exact value of the NS form ellipticity ϵ and frequency and can be detected by advanced LIGO/VIRGO interferometers. In reality, the magnetic field of NS causes more effective electromagnetic energy loss: to be insignificant, the magnetic field of a NS should be less than (see Eq. (9)) $\mu < 1.5\times 10^{26}(\text{G cm}^3)\epsilon_{-6}\, f$

According to Urpin & Muslimov (1992), the magnetic field can decay very fastly provided that the field was initially concentrated in the outer crust layers with the

density $< 10^{10} - 10^{11}$ g cm^{-3}, and such very low magnetic field for old NS may be possible. In the limiting case that the NS magnetic field does not decay at all (for example, if only accretion-induced field decay is possible in binary systems (Bisnovatyi–Kogan & Komberg 1974)), old NS should lose their energy through electromagnetic losses and be very slow rotators with periods of about a few seconds. Then the initial magnetic field distribution becomes crucial. If it is centered at $\sim 10^{12}$ G (as implied by radipulsar P–$\dot{P}$ measurements), we have little chances to detect the old NS population at 10–100 Hz frequency band unless close mean distances (<10 kpc) are assumed (Giazotto et al. 1997). However, if nature prefers a scale-free law (like $f(\mu) \propto 1/\mu$), the fraction of low-field NS could amount to a few 10% and they can contribute to the frequency-independent GR background. Then Eq. (8) implies that such a background can be detected by the advanced LIGO/VIRGO interferometer in the frequency band 10–1000 Hz in one-year integration even if the formation rate of such NS is as small as 1 per 300 years and the characteristic distance to them is 100 kpc.

4 GW noise from unresolved binary stars at LISA frequencies

Inside LISA frequency range, $10^{-4} - 10^{-1}$ Hz, only coalescing binary white dwarfs (WD) and binary neutron stars contribute. Even if binary neutron stars coalesce at a rate of 1/10000 yr in the Galaxy, their number still should be much smaller than the white dwarf binaries, and in this section we restrict ourselves to considering only binary WD.

Substituting $E = E_{orb} \sim \mathcal{M}c^2(\mathcal{M}f)^{2/3}$ into equation (6) we obtain

$$\Omega_{WD}(f) \approx 2 \times 10^{-8} \mathcal{R}_{100} (f/10^{-3}\mathrm{Hz})^{2/3} (\widetilde{\mathcal{M}}/M_\odot)^{5/3} (\langle r \rangle / 10\,\mathrm{kpc})^{-2} h_{100}^{-2}\,, \tag{11}$$

where $\mathcal{R}_{100} = \mathcal{R}/(0.01\ \ \mathrm{yr}^{-1})$ is the galactic rate of binary WD mergers.

In terms of the characteristic dimensionless amplitude of the noise background that determines the signal-to-noise ratio when cross-correlating outputs of two independent interferometers we have

$$h_c(f) \approx 7.5 \times 10^{-20} \mathcal{R}_{100}^{1/2} (f/10^{-3}\mathrm{Hz})^{-2/3} (\widetilde{\mathcal{M}}/M_\odot)^{5/6} (\langle r \rangle / 10\,\mathrm{kpc})^{-1} \tag{12}$$

Equation (12) shows that at high frequencies of interest here the GW background is fully determined by the galactic rate of binary WD mergers and is independent of (complicated) details of binary evolution at lower frequencies (the examples of calculated spectra at lower frequencies see in Lipunov & Postnov 1987; Lipunov, Postnov & Prokhorov 1987; Hils et al. 1990).

But the real galactic merger rate of close binary WD is unknown. One possible way to recover it is searching for close white dwarf binaries. A recent study (Marsh

et al. 1995), revealed a larger fraction of such systems than had previously been thought. Still, the statistics of such binaries in the Galaxy remains very poor.

If coalescing binary WD are associated with SN Ia explosions, as proposed by Iben & Tutukov (1984) and further investigated by many authors (for a recent review of SN Ia progenitors see Branch et al. 1995), their coalescence rate can be constrained using much more representative SN Ia statistics. Branch et al. (1995) concluded that coalescing CO-CO binary WD remain the most plausible candidates mostly contributing to the SN Ia explosions. The galactic rate of SN Ia is estimated 4×10^{-3} per year (Tamman et al. 1994; van den Bergh and McClure 1994), which is close to the calculated rate of CO-CO coalescences ($\sim (1-3) \times 10^{-3}$). The coalescence rate for He-CO WD and He-He WD (other possible progenitors of SN Ia) falls ten times short of that for CO-CO WD (Branch et al. 1995). As SN Ia explosions may well be triggered by other mechanisms, we conclude that the observed SN Ia rate provides a secure *upper limit* to the double WD merger rate regardless of the evolutionary considerations.

The upper limit (12) is plotted in Fig. 1 for different rates of binary WD mergers $\mathcal{R}_{100} = 1, 1/3, 1/10, 1/30$ assuming the chirp mass $\mathcal{M} \approx 0.52 M_{\odot}$ (as for two CO white dwarfs with equal masses $M_1 = M_2 = 0.6 M_{\odot}$). These lines intersect the proposed LISA rms sensitivity at $f > f_{lim} \approx 0.03 - 0.07\text{Hz}$. This means that at frequencies higher than 0.07 Hz no continuous GW backgrounds of galactic origin are presently known to contribute above the rms-level of LISA space laser interferometer. The contribution from extragalactic binaries is still lower regardless of the poorly known binary WD merging rate (at least in the limit of no strong source evolution with z). Other possible sources could be extragalactic massive BH binary systems (e.g. Hils and Bender 1995). Their number in the Universe can be fairly high (e.g. Rees 1997), but no reliable estimates of their contribution are available at present. The lower limit (4) is already close to the LISA sensitivity limit at 0.1 Hz, but we stress that the assumptions used in its derivation are upper limits, so the actual frequency beyond which no binary stochastic backgrounds contribute may be three times lower. This precise limit depends on the details of binary WD formation and evolution which are still poorly known.

Fig. 1 demonstrates that the calculated GW background intersects LISA sensitivity curve at frequencies ~ 0.05 Hz, and Bender et al's curve at even lower frequencies ~ 0.01 Hz. The latter is probably due to Bender et al's curve being derived from observational estimate of double WD galactic density in the solar neighborhoods; we stress once more that once formed, the binary WD will evolve until the less massive companion fills its Roche lobe; unless the mass ratio is sufficiently far from one (cf. Webbink 1984), the merger should occur. Therefore Bender et al's curve provides a secure *lower limit* to the galactic binary stochastic GW background.

Presently, we cannot rule out the high galactic double WD merger rate (1/300-1/1000 yr^{-1}), and therefore can consider f_{lim} to lie within the frequency range $0.01 - 0.07$ Hz. We conclude that no GW background of galactic origin above this

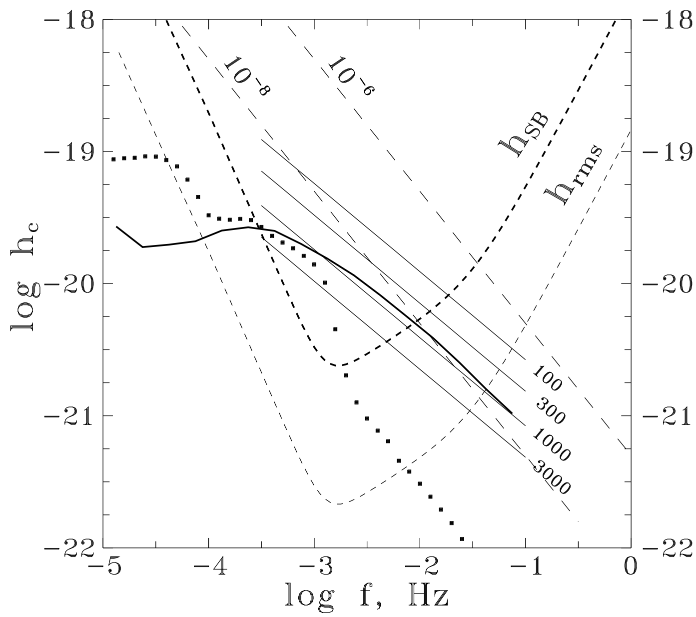

Figure 1: Galactic binary GW background h_c as given by Bender (1996) (filled quadrangles) and calculated for a model spiral galaxy with the total stellar mass 10^{11} $M_\odot$ (the solid curve). Average photometric distance 7.9 kpc is assumed. Thin straight lines marked with 100, 300, 1000, 3000 are the analytical upper limit (eq. [12]) for binary WD merger rates 1/100, 1/300, 1/1000, and 1/3000 yr^{-1} in a model spiral galaxy, respectively, assuming $\mathcal{M} = 0.52$ $M_\odot$. Straight dashed lines labeled by 10^{-8}, 10^{-6} show GW backgrounds corresponding to constant Ω_{GW}. The proposed LISA rms noise level (h_{rms}) and sensitivity to bursts $h_{SB} = 5\sqrt{5}h_{rms}$ are also reproduced (cf. Thorne 1995; Fig. 14).

frequencies should contribute at the rms-noise level of LISA interferometer, and hence the detection of an isotropic stochastic signal at frequencies $0.03-0.1$ Hz with an appreciable signal-to-noise level (which possibly may be done using one interferometer) would strongly indicate its cosmological origin. To be detectable by LISA, the power of relic GW background should be $\Omega_{GW}h_{100}^2 > 10^{-8}$ in this frequency range.

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